References

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Snowing Criteria for Cold Traps

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Introduction

A COLD trap is a flow channel with cold walls. When a mixture of condensable and noncondensable gases flows between the cold walls, the condensable species diffuse to the walls where they are trapped by condensation and perhaps frozen. In oxygen-iodine lasers, cold traps are used to remove water vapor from an excited oxygen flow. The oxygen is produced in a reaction including an aqueous solution, which leads to water vapor in the flow. This water vapor must be largely removed, because it causes undesirable reactions in the laser cavity. Typically, a maximum water vapor partial pressure requirement is specified; the cold trap surfaces must be cold enough so that the ice on these surfaces has a vapor pressure significantly below the partial pressure requirement. The channel dimensions are determined by considering mass transfer efficiency, gas residence time compared to excited oxygen deactivation times, and pressure drop.

Cold traps also act as heat exchangers and cool the gas as condensable species are removed. The gas cooling tends toward supersaturating the gas, while condensable species removal has the opposite effect. This tradeoff of the local concentration or partial pressure of, say, water vapor p_{H_2O} and the local temperature T can be seen in the supersaturation parameter

$$S = p_{H_2O}/p_{sat}(T)$$

where $p_{sat}(T)$ is the saturation pressure of water, which is correlated (in Pa and K) here as

$$p_{sat}(T) = 133.3\exp(24.1 - 6165/T)$$

When the gas mixture exceeds saturation conditions, fog or ice particles can form in the gas away from the walls. "Heterogeneous nucleation," where water condenses on existing aerosols, occurs with even slight supersaturation, $S>1$. "Homogeneous nucleation" or "spontaneous nucleation" of new fog or snow particles requires greater supersaturation, $S>S_c$, where S_c is the critical supersaturation. In either case, snow is untrapped water that can revaporize in a hot laser cavity.

Temperature and Concentration Profile Analogy

Temperature and concentration gradients are larger in the cross-stream directions than they are in the flow direction. A narrow region in the entrance boundary layer near the channel wall could supersaturate sufficiently to snow, while the gas remains unsaturated at the channel centerline.

An analogy between temperature profiles and concentration profiles will allow a determination of peak supersaturation without actually computing these profiles themselves. The local dimensionless gas temperature is defined as

$$\theta = (T - T_w)/(T_0 - T_w)$$

where T_w is the channel wall temperature and T_0 is the gas inlet temperature. The total pressure in a cold trap typically changes very little, so that the normalized water mole fraction is the same as the dimensionless water partial pressure

$$\phi = \frac{x_{H_2O} - x_{H_2O,w}}{x_{H_2O,0} - x_{H_2O,w}} = \frac{p_{H_2O} - p_{H_2O,w}}{p_{H_2O,0} - p_{H_2O,w}}$$

Equilibrium at the wall demands $p_{H_2O,w} = p_{sat}(T_w)$. Both θ and ϕ vary continuously from 0 to 1 regardless of the channel geometry and regardless of whether the flow is laminar or turbulent. Both θ and ϕ are zero at the wall and far downstream and are unity in a short region near the channel entrance at the channel centerline—they have the same boundary conditions.

The energy conservation equation describing θ and the molar species conservation equation describing ϕ are exactly analogous with the Prandtl number Pr and the Schmidt number Sc, being analogous parameters (see any basic heat and mass transfer text[1]). Define a parameter C by $\phi = C\theta$. If $Pr \simeq Sc$, as for many gases, then $C \simeq 1$. Also, for turbulent flow, eddy diffusivities are the same for both the energy and the molar species equations, and these eddy diffusivities dominate over molecular diffusivities, so that again $C=1$. Laminar thermal boundary-layer thicknesses are approximately proportional to $Pr^{-0.4}$ (see Ref. 2), so that for laminar boundary layers near the trap inlet $C \simeq (Sc/Pr)^{0.4}$, which for O_2 and H_2O is $C=0.95$.

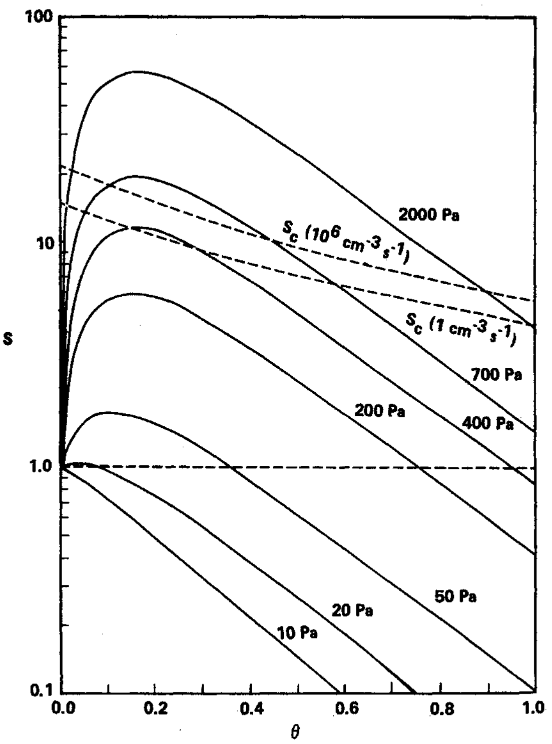

Fig. 1 Supersaturation profiles as a function of θ for various values of p_0^*, the inlet water partial pressure ($T_w = 218$ K, $T_0 = 270$ K).

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Heterogeneous Nucleation

The relation of temperature to partial pressure through θ is useful in computing the supersaturation parameter. The supersaturation parameter can now be expressed as

$$S=\frac{p_{H_2O}}{p_{sat}(T)}=\frac{p_{sat}(T_w)+\theta[p_0^*-p_{sat}(T_w)]}{p_{sat}[T_w+\theta(T_0-T_w)]}$$

where

$$p_0^*=Cp_{H_2O,0}+(1-C)p_{sat}(T_w)\simeq p_{H_2O,0}$$

Note that often $C\simeq 1$ and also $p_{sat}(T_w)\ll p_{H_2O,0}$. Thus the supersaturation parameter is expressed in terms of known constants (T_0, T_w, and $p_{H_2O,0}$) and one variable θ. The dimensionless temperature θ can be thought of as the distance from the wall into the flow because it is zero at the wall and rises monotonically to a maximum value (less than or equal to one) at the channel centerline. The supersaturation parameter as a function of θ is plotted in Fig. 1 for various inlet water partial pressures with a 270 K inlet gas temperature and a 218 K trap surface temperature. The gas is always saturated ($S=1$) right at the surface ($\theta=0$). For low initial water vapor levels, the supersaturation parameter is less than one at all points away from the wall and there is no snow of any kind. With larger initial water amounts, the supersaturation parameter rises to a peak and then falls off further from the wall. This peaking always results in a band of θ values next to the cold trap surface where the supersaturation parameter is greater than one, and where water vapor can heterogeneously condense on existing aerosol particles. The criterion for avoiding heterogeneous nucleation in the gas flow (avoiding $S>1$) is a negative slope of S at the wall.

$$\left.\frac{dS}{d\theta}\right|_{\theta=0}<0$$

which is the same as

$$p_0^*<[1+6165(T_0-T_w)/T_w^2]p_{sat}(T_w)$$

The inlet gas supersaturation parameter is S evaluated at $\theta=1$. The 50 Pa inlet water partial pressure case shown in Fig. 1 is an example where the inlet gas is an order of magnitude below saturation ($S=0.1$), yet a portion of the gas flow near the wall is supersaturated. Water vapor partial pressures must be extremely low to avoid heterogeneous nucleation.

Homogeneous Nucleation

Homogeneous nucleation could result in substantial amounts of snow even in a perfectly clean gas flow. Homogeneous nucleating occurs for supersaturation above a critical level S_c. Katz[3] found critical supersaturation criteria as a function of gas temperature, which are correlated here as

$$S_c=\exp(1.276-991/T+2.813\times 10^5/T^2)$$

for the 1 nucleus/cm$^3\cdot$s rate, as

$$S_c=\exp(1.347-1108/T+3.231\times 10^5/T^2)$$

for the 10^6 nuclei/cm$^3\cdot$s rate, and as

$$S_c=\exp(1.536-1324/T+3.928\times 10^5/T^2)$$

for the 10^{12} nuclei/cm$^3\cdot$s rate. The 1/cm$^3\cdot$s and 10^6/cm$^3\cdot$s critical supersaturations (Fig. 1) are within a factor of about 1.5 of each other, which is very close considering the 10^6 factor difference in the nucleation rate. Much higher initial water vapor levels are needed to exceed the homogeneous nucleation criteria ($S=S_c$) than are needed to exceed the heterogeneous nucleation criteria ($S=1$). When the supersaturation parameter peak crosses over the critical supersaturation curve, there is a band of θ values (those where $S>S_c$) where homogeneous nucleation will occur. When this band of θ values is wide, as for the 2000 Pa initial water partial pressure case shown in Fig. 1, homogeneous snow formation will occur in almost the entire channel. Homogeneous nucleation can be avoided in the design and operation of cold traps, however, by keeping below the "critical" initial partial pressure of water vapor, where the S curve just contacts the S_c curve. This critical pressure is a function of the wall temperature, so from a practical standpoint controlling the wall temperature is the way to avoid snowing.

The critical initial partial pressure of water vapor for homogeneous nucleation (for given values of T_w and T_0) would be that for which the supersaturation profile as a function of θ is tangent to the critical supersaturation curve. This critical pressure can be computed by fixing T_0 and T_w while solving the two coupled nonlinear equations,

$$f(\theta,p_0^*)=S-S_c=0,\qquad g(\theta,p_0^*)=\frac{dS}{d\theta}-\frac{dS_c}{d\theta}=0$$

for the two unknowns, θ and p_0^*. A two-dimensional Newton-Raphson iteration procedure was used to solve these equations for several T_0 values and as a function of T_w, as plotted in Fig. 2 for 1 nucleus/cm$^3\cdot$s S_c. Partial pressures above these homogeneous snow criteria curves result in snow or, moving horizontally, trap surface temperatures less than those indicated by the curves (at fixed initial water partial pressure) result in snow. The analytical heterogeneous snow criteria are also plotted in Fig. 2 and can be interpreted similarly. The vapor pressure of the ice on the cold trap surfaces is plotted in Fig. 2; for initial water pressures below this wall saturation level, there is no water trapping and no purpose for a trap.

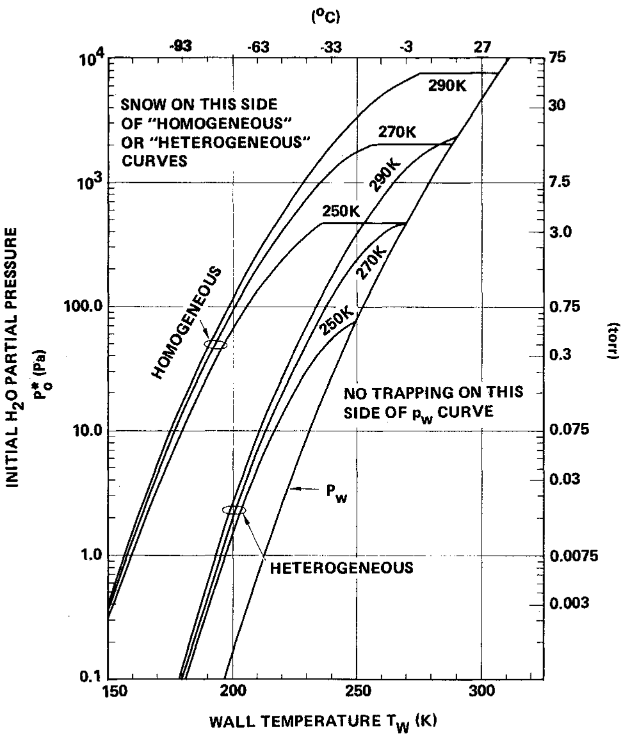

Fig. 2 Onset of homogeneous and heterogeneous nucleation at 1 nucleus/cm$^3\cdot$s as a function of trap wall temperature for different inlet gas temperatures.

Conclusions

Heterogeneous nucleation is seen to begin at partial pressures of about 10 times the wall partial pressure. Homogeneous nucleation is seen to begin at partial pressures on the order of 10^3 times the wall partial pressure. Initial gas temperature has only a weak effect on the onset of snow as long as the wall temperatures are substantially below this inlet temperature. Snow may be avoided by staying below the onset curves, lowering the inlet partial pressures of the water vapor, or increasing the wall temperatures. These calculations and curves apply to any uniform temperature water cold trap. These curves can even be used to predict the humidity required for the thin natural convection boundary layer flowing down the outside of a glass of ice water to be visible due to small droplet formation (fog) in this boundary layer.

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Engine Power Simulation for Transonic Flow-Through Nacelles

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I. Introduction

EXISTING inlet and nacelle analyses examine either "isolated inlets," i.e., semi-infinite nacelles with prescribed mass flux, or finite length "flow-through" nacelles, where Kutta's condition determines the mass flow. For example, panel methods, finite difference and element solutions, and Euler and Navier-Stokes solvers presently are available for engineering analysis.[1-4] This Note addresses the effect of jet engine power addition. The interaction between the external potential flow past a finite length nacelle and an internal irrotational flow with increased total pressure is discussed in the transonic small disturbance limit (this interaction occurs at the actuator disk, where flow properties change abruptly, and through the trailing-edge slipstream where significant velocity discontinuities are found). A simple, physically ratational and easily implementable numerical model requiring only minor modification to existing airfoil codes is developed for use in preliminary design.

This work expands on the author's earlier research. In Ref. 5, a finite length nacelle is immersed in a uniform, constant density potential flow; a radial parallel shear flow is assumed at the actuator disk, producing a rotational flow which interacts with the external flow through the downstream plume. This rotational flow was solved using a "superpotential" ϕ^*. Extended Cauchy-Riemann conditions for ϕ^* were inferred from a linearized disturbance streamfunction equation cast in conservation form; ϕ^* then satisfies a simple "potential-like" equation. Invariance properties of vorticity were next used to rewrite Bernoulli's equation so that wake pressure continuity could be expressed using jumps in ϕ^*. Finally, tangency conditions, expressed by normal derivatives, result in an easily implementable formulation (the general approach and numerical results for a strong shear appear in Ref. 5). This approach does not handle compressibility. In Ref. 6, a transonic irrotational freestream producing shock waves on the external nacelle surface was assumed; for simplicity, a radially uniform increase in total pressure was prescribed so that the internal flow remains potential. Reference 6 discusses the nontrivial details required to computationally match different potential equations referenced to different thermodynamic conditions and provides results of sample calculations.

When the latter work was completed and satisfactorily evaluated, it was evident that the complexities introduced to ensure thermodynamic consistency were not necessary in view of the small disturbance equation used. The aim then focused on a simple preliminary design tool employing approximations consistent with transonic small disturbance theory. The basic idea is simple. Airfoil analysis, for example, solves the same potential equation fore and aft of any calculated shocks, expanded about the same freestream Mach number, despite a discontinuous change in local Mach number; in our engine-nacelle problem, where the planar equation would be modified by an axisymmetric ϕ_r/r term, we will assume a change in reference Mach number through the actuator disk of, at most, the same order. The mathematical model is discussed next.

II. Analytical and Numerical Approach

Let $\phi_1(x,r)$ be the external disturbance potential, x and r being streamwise and radial coordinates, and, assuming a constant total pressure increase, let $\phi_2(x,r)$ represent the

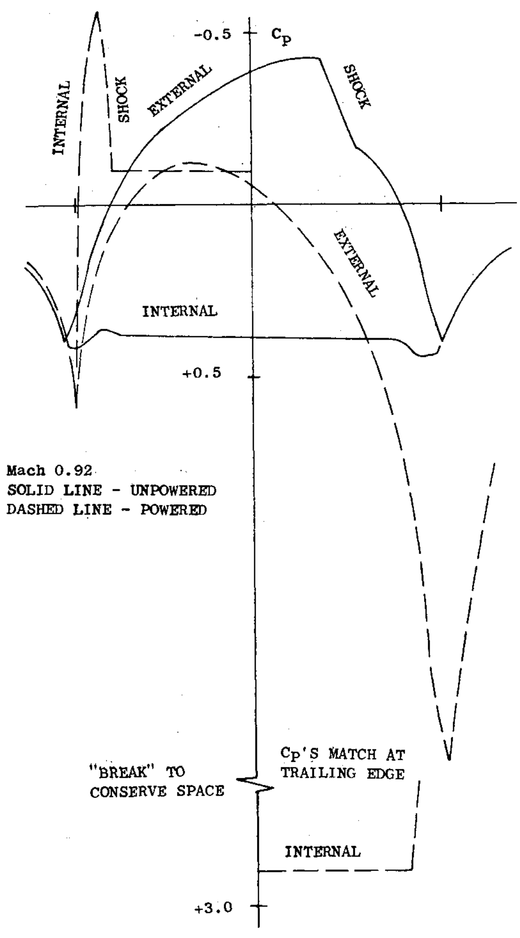

Fig. 1 Mach 0.92 results.

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